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Geometry texts: a different look for our times

Los textos de geometría: una mirada diferente para nuestros tiempos

Textos geométricos: um olhar diferente para nosso tempo.

Research Article

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Abstract

Introduction: The function of textbooks as a didactic resource and the impact they have on mathematics education, particularly geometry, makes it pertinent to carry out hermeneutic and critical analyzes of the characteristics and adaptations to reality. current Colombian. **Objective:** This research seeks to study mathematics school textbooks, in which we find the subject of geometry immersed, through the observation of the structure they use, in such a way as to verify if they contribute to the generation of significant learning. in the student body, and not simply become a continuation of classes. **Method and/or methodology:** A content analysis design was used as a technique for the objective, systematic and qualitative description of the manifest content of the communications, in order to interpret it. The most relevant results show the need for school textbooks to point to digital texts where the resolution of the deficiencies presented in current texts is focused. **Conclusion:** through the digitization of geometry texts, it is possible to contribute to the improvement of visualization through interactive and dynamic environments of more assertive content in the area.

Key words: Geometry; School texts; Technology and innovation; Digitization.

Resumen

Introducción: La función de los libros de texto como recurso didáctico y el impacto que tienen éstos sobre la educación matemática, en particular de la geometría, hace que resulte pertinente la realización de análisis de tipo hermenéutico y crítico sobre las características y adaptaciones a la realidad actual colombiana. **Objetivo:** La presente investigación busca estudiar los textos escolares de matemática, en los que encontramos inmersa la temática de la geometría, a través de la observación de la estructura que utilizan, de tal manera que se verifique si contribuyen a la generación de un aprendizaje significativo en el estudiantado, y no simplemente se convierten en una continuación de las clases. **Método y/o metodología:** Se trabajó un diseño de análisis de contenido como técnica para la descripción objetiva, sistemática y cualitativa del contenido manifiesto de las comunicaciones, con el fin de interpretarla. Los resultados más relevantes muestran la necesidad de que los textos escolares le apunten a textos digitales donde se enfoque a la resolución de las deficiencias presentadas en los textos actuales. **Conclusión:** a través de la digitalización de los textos de geometría se puede contribuir a la mejora de la visualización mediante entornos interactivos y dinámicos de contenidos más asertivos en el área.

Palabras Clave: Geometría; Textos escolares; Tecnología e innovación; Digitalización.

Resumo

Introdução: O papel dos livros didáticos como recurso didático e o impacto que têm na educação matemática, particularmente na geometria, torna pertinente a realização de uma análise hermenêutica e crítica das características e adaptações à realidade colombiana atual. **Objetivo:** A presente pesquisa busca estudar os livros didáticos de matemática, nos quais encontramos o tema da geometria imerso, através da observação da estrutura que utilizam, de forma a verificar se contribuem para o a geração de aprendizado significativo nos alunos, e não se tornar simplesmente uma continuação das aulas. **Método e/ou metodologia:** Um projeto de análise de conteúdo foi utilizado como técnica para a descrição objetiva, sistemática e qualitativa do conteúdo manifesto das comunicações, a fim de interpretá-lo. Os resultados mais relevantes mostram a necessidade de que os livros escolares visem textos digitais que focalizem a solução das deficiências para a resolução das deficiências apresentadas nos textos atuais. **Conclusão:** através de a digitalização de textos de geometria pode contribuir para a melhoria da visualização por meio de uma através de ambientes interativos e dinâmicos de conteúdo mais assertivo na área.

Palavras-chave: Geometria; Textos escolares; Tecnologia e inovação; Digitalização.

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Introduction

Since their appearance, school books or textbooks have represented an extremely important reference in the transmission of knowledge. Thus, Chopin (1980) quoted by Emmanuele, González, Introcaso and Braccialarghe, considers that the textbook is:

At the same time, it supports knowledge in that it imposes a distribution and hierarchy of knowledge and contributes to forging the intellectual scaffolding of both students and teachers; it is an instrument of power, since it contributes to the linguistic standardization of the discipline, to cultural leveling and to the propagation of dominant ideas (2010, p. 2). (2010, p. 2).

In this sense, it is pertinent to analyze textbooks from their function as a didactic resource and as a source of knowledge, with a marked socio-cultural component that has had an impact on mathematics education.

In the case of the analysis of geometry textbooks in the currents of mathematics teaching, it is mentioned that historically the entry of these texts into teaching was carried out from the French school, which was favored by the political and historical context of the United States; it was done with the purpose of improving education by introducing modern mathematics in schools, and, therefore, texts where geometry was immersed. Through the institutionalization of the texts, it was possible to count on financial contributions that made it possible to form working groups for the elaboration of school texts, teachers' guides and teacher training (García, 1996).

For decades, certain movements have arisen that in some way have led to the reflection, modification and elaboration of didactic materials, texts or manuals; this is especially relevant, as Cantoral (cited by García, 1996) points out. Other research referred to by García analyzes the didactic changes produced by the systematic use of translated texts. The results show that teachers continue to use school texts from the first half of the century. Finally, it can be said that the school mathematical discourse as a discourse that, from educational scenarios, particularly in Latin America, opens the possibility of participating in the redesign of mathematical knowledge through the coverage of the epistemological, didactic and cognitive components.

As this is a documentary study, we intend to carry out a hermeneutic and critical analysis of geometry textbooks, which will allow us to interpret and understand their characteristics and relevance. This will enable us to review or take stock of the new scenarios that may arise in relation to the topic in question, taking into account the current situation and the progress made in the field of this type of research on this subject.

All of the above, generated a questioning about the management and knowledge of the historical and pedagogical evolution of geometry contents in mathematics textbooks; making it possible to have a conceptual, curricular and didactic background of what has been the research work in Colombia regarding this particular subject. Knowing its history allows to facilitate the understanding of the currents and categories, offering a constant and systemic reflection of the reforms carried out.

GEOMETRY TEXTS A PREDOMINANT TOOL IN THE CLASSROOM.

Given that the main focus of this analysis is directed to the characterization of the contents of primary school textbooks, mainly framed in geometry, it will be necessary to raise some concepts that will allow us an interpretative reading.

Research on textbooks has become an effective method for the study of teaching and learning processes. As it is well known, textbooks have a great influence in the classroom; therefore, it is very important that mathematics teachers and researchers in mathematics education analyze textbooks from the didactic point of view. Thus, as Espinoza, Pochulu and Jorge mention: "Didactic analyses of texts allow characterizing the quality of the textualized mathematical organization, its degree of completeness, pertinence, adequacy, and epistemic and didactic suitability" (2013, p. 1).

In this way, the textbook represents in the classroom the reference material, the practice exercises for the student, a guide for the organization of the curriculum, among others, thus fulfilling an influential role in school practices. Hence, in mathematics education there are several approaches and lines that show didactic analyses of textbooks, which highlight the strength and receptiveness of the tools used.

As an example of this we have: Godino, Font and Wilhelmi (2006), who explain the technique of analysis of mathematical texts from the onto-semiotic approach of mathematical cognition and also propose a concept of global suitability from 5 criteria of suitability in the teaching-learning process, this means for the teacher a valuable tool in their training to establish criteria for improvement.

At the same time, other research has been supported by the new theoretical development, allowing us to test its relevance and applicability. Among these we can highlight Etchegaray (2001), who uses the semiotic-anthropological approach to analyze the most widely circulated books and finds in-compatibility between institutional and personal meanings.

Likewise, Cantoral, Montiel and Reyes-Gasperini (2015), suggest that the analysis of textbooks can be carried out from the socio-epistemological theory of Educational Mathematics, in which the social environment plays an important role from the problematization of knowledge and deepening in the school mathematical discourse.

In this sense, textbooks have historically influenced the development of education and are interesting to investigate since they allow us to study the different approaches and changes under which a discipline or concept has been approached. We can then bring up some research such as that of Schubring (1987), who analyzes historical mathematics texts, through which he can extract information on the evolution of knowledge through the different eras and the relationship or influence between cultural and political processes with the teaching-learning processes.

These types of relationships or social interactions were also of interest to Radford (1997) and Otte (1997) who consider that from the socio-cultural perspective, knowledge is obtained from social activity and that the history of mathematics contributes much to the epistemology of mathematics, this science being a social construct.

In this same context, the a priori and a posteriori comparative study carried out by How- son (1995) on textbooks from different countries stands out, indicating the contributions that a book has made, after its use, in the teaching-learning process and the cultural influences of each country in these processes. Likewise, there is the work of Chevallard and Joshua (1982), in their article "Un exemple d'analyse de la transposition didactique: La notion de distance" in which they use the notion of didactic transposition related to the transformations between the knowledge known and the knowledge taught and between which there is an intermediate step corresponding to the knowledge to be taught that is reflected in the text of the knowledge.

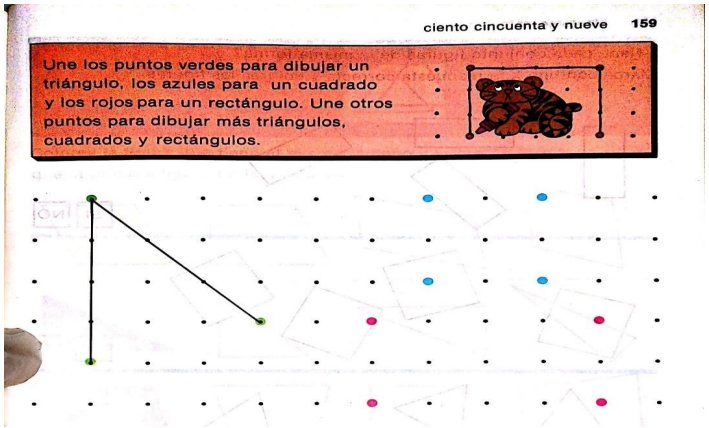
Other research focuses on aspects related to the language and readability of the texts Pimm (1987) and Pimm and Keynes (1994) or on the form of presentation of the contents, such as that of Otte (1997), who emphasizes what the text conveys, the relationships between knowledge and textual representation, and variations in interpretations. In turn, Van Dormolen (1986) classifies the elements that are essential in a mathematics textbook, and Love and Pimm (1996) consider that there is a tetrad associated with a textbook: the reader, the writer, the teacher and the book itself, and that the characteristics of each of them, as well as their interactions, determine the use of this material in the classroom.

For their part, Rodríguez, Basso and García (2019) consider that the analysis of textbooks contributes to the understanding of problems in the field of mathematics education research since it allows improving processes and learning.

In the first instance, there is the book Mathematics for Basic Education 2 published by Editorial Norma in 1980, aimed at students in the second grade of elementary school. Regarding the subject of geometric figures, it can be mentioned that the information contained in the text is oriented to the solution of problems or to answer questions by making inferences, after performing exercises where motor skills are tested; for example: "Join the green dots to draw a triangle, the blue ones to draw a square, the red ones to draw a rectangle. Join other dots to draw more triangles, squares and rectangles". These problems are expressed as text boxes as shown in Figure 1.

Figure 1.

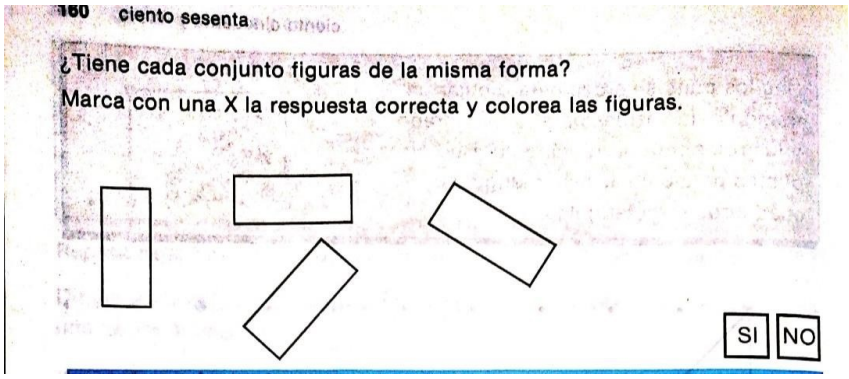
Motor activity.



Source: Figure elaborated based on the book Mathematics for Basic Education 2.

Then a question is posed to verify if the student identifies the types of figures; this time through groups, the question is "Does each group have figures of the same shape? With a closed answer YES or NO, as shown in figure 2.

Figure 2.
Verification activity 1.

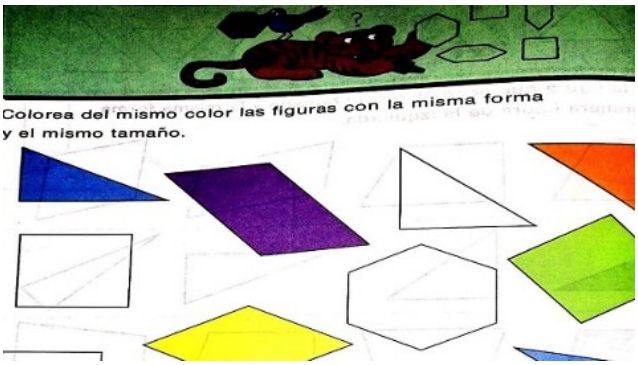


Source: Figure elaborated based on the book Mathematics for Basic Education 2.

The above indicates that this textbook seeks that the student reinforces previous knowledge about geometric figures. It should be mentioned that the book contains few activities where aspects of the student's daily life are related; likewise, the definitions within the book are scarce, and notes are visualized around the exercises. Furthermore, the activities do not mention objects or images that represent or contain shapes similar to geometric figures. In addition to the above, there are no tables in the textbook.

As for the socio-cultural aspect, it can be noted that the curricular considerations of the time are oriented in the book because it was produced by Editorial Norma, which distributes a large number of copies throughout the country. In other words, the expressions of the exercises and other activities are adapted to the requirements of the Ministry of National Education (MEN). The representation of the figures is simple and printed in the foreground, with ample space for coloring and scratching, as shown in the figure:

Figure 3.
Verification activity 2.



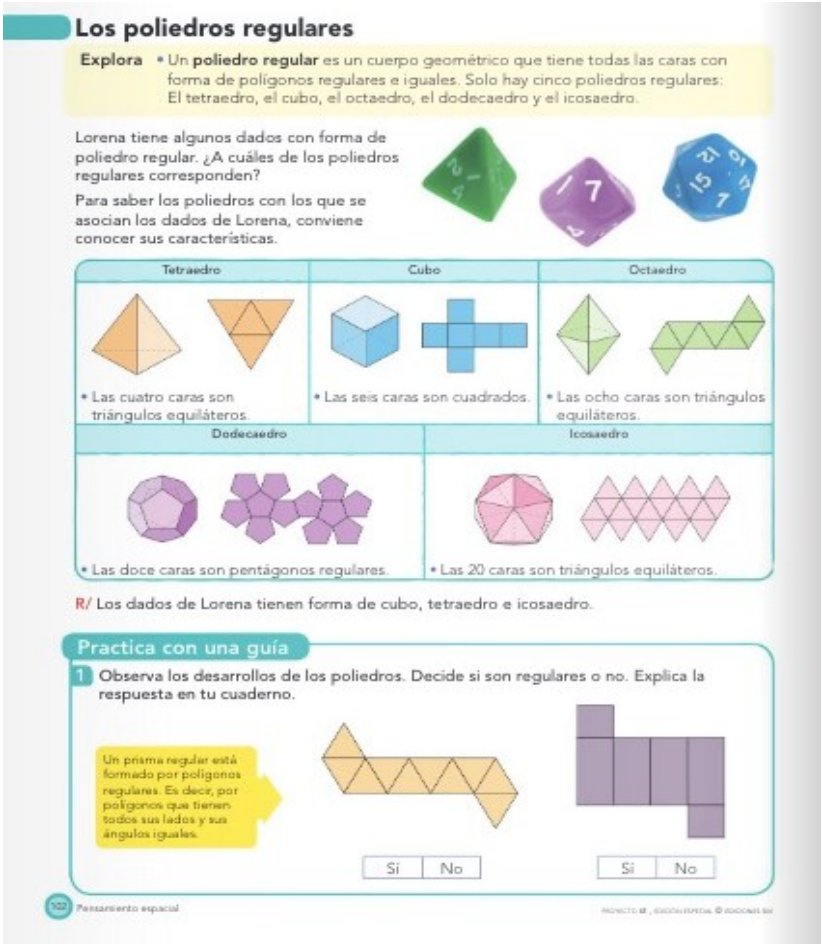
Source: Figure elaborated based on the book Mathematics for Basic Education 2.

Table 1.
Analysis of the book: Mathematics for Basic Education 2

Categories	Dimensions	Exhibition	Technological	Comprehensive
Syntax	Structure of the problem		*	*
	Theoretical descriptions		*	*
	Symbols used in the graphs			
	Types of expressions		**symbolic**	*
Semantics	Phenomenology	*	*	
	Types of graphs	* Types of **descriptions**		
Pragmaticdidactics	Meaning of symbolic expressions			*
	Function of the exercises		*	*
	0 Role of definitions		*	
	1 Graphic activities		*	*
	2 Role of symbolic expressions		*	*
Sociocultural	3 Social influence and adaptation to the curriculum			*
	4 Didactic influences		*	*
	5 Presentation of graphs (static/dynamic)		*	
	6 Complexity of symbolic expressions.		*	*

Source: Own elaboration.

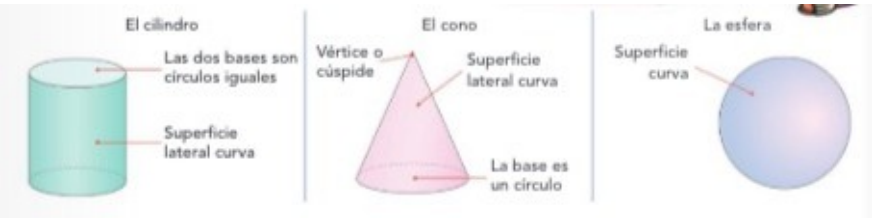
Figure 3.
Verification activity 2.



Source: Figure elaborated based on the book Mathematics for Basic Education 2.

The Pedagogical Component of the Program proposes diverse, multiple, permanent and sustained interactions among the different actors of the educational institution in democratic environments that recognize the socioeconomic and cultural contexts in which children develop and the joint purpose of consolidating a pedagogical project that offers better opportunities, transforms their objective living conditions and develops their capabilities. For this purpose, the graphic material is supported by photographs of elements of the modern world such as wind generators, windmills, among others.

Figure 5.
Geometry



Source: Figure elaborated based on the book Mathematics for Basic Education 2.

Book National Government Project

Taking into account the categories of analysis in terms of syntax, semantics, pragmatics -didactics, and socio-cultural, the book Project I know how to be technological can be profiled. Although, some dimensions such as phenomenology, types of descriptions, the function of exercises and graphic activities, share a tendency towards expository typology, as shown in Table 2.

Table 2.
Analysis of the book: Project I know

Categories		Dimensions	Exhibition	Technological	Comprehensive
	1	Structure of the problem		*	*
	2	Theoretical descriptions		*	
Syntax	3	Symbols used in the graphs	*	*	
	4	Types of symbolic expressions		*	
	5	Phenomenology		*	*
	6	Types of descriptions		*	
Semantics	7	Types of graphs			*
	8	Meaning of symbolic expressions			*
	9	Function of the exercises		*	*
	10	Role of definitions			*
Pragmaticdidactics	11	Graphic activities			*
	12	Role of symbolic expressions			
	13	Social influence and adaptation to the curriculum			
	14	Didactic influences	*		
Sociocultural	15	Presentation of graphs (static/dynamic)			
	16	Complexity of symbolic expressions			*

Source: Own elaboration.

Likewise, the exercise was done with books such as: Nacho from Susaeta publishing house, Vivamos las Matemáticas serie 4, Alegría de las Matemáticas editorial REI Andes LTDA, Eureka 3° editorial McGraw Hill, Cuenta Jugando 3° editorial Norma group, Nova editorial Voluntad S.A, Matemáticas, Aplicaciones y Conexiones level 4 editorial McGraw Hill, Serie de Matemáticas para Educación Básica Primaria from Norma publishing house, Libro de los Números editorial Klet, among others.

It is an undoubted theory that the mathematics textbook is the main resource to explain and plan problems and exercises, but it is also the axis that constitutes the subject. The strong influence exerted by the use of a textbook and the ways in which it can be used without distinction in culture or socioeconomic status is framed, as a consequence of its structure, classification and diverse approaches, as well as the construction of the categories of analysis which are a complex process based on the bibliographic review, as well as on the projected goals of the research.

With respect to the identification of the categories of analysis for textbooks or school textbooks, we can say that these categories denoted changes and as a result we were able to determine categories as a whole with appropriate elements to study each one of them: description, criteria, subcatego- ries, tables and graphs for the collection and presentation of the data. These were grouped into five categories of analysis, which we finally arrived at: Presentation of figures and geometric concepts, Flat representation of three-dimensional figures, Orientation distracters, Structuring distracters, Real images of figures and geometric concepts.

At this point it is important to emphasize that many texts are based on some models of geometry teaching such as Duval, Van Hiele and Rene Descartes, who is said to be the birth of the new geometry: Analytical Geometry, which was made known in England by John Wallis with his work "Trac- tatus de sectionibus conicis", and in Holland by Franz van Schooten with his "Comment!"to Descartes' "Geometry", nowadays the importance and the use of the texts that were officially elaborated by Prometam for the Ministry of Education is shown.

These texts bring together all the influences on which the history of geometry and its teaching is based, which determine the most relevant content to apply in the study of geometry and which will also allow the construction of problem solving, giving the teacher the tool that allows him to develop a class where the objectives are met, all the content is given and the reasoning of the students is developed.

One of the most widely used didactic tools is the textbook; therefore, the study of textbooks as a source of learning continues to be an interesting world to discover and of great importance for the quality of our educational system. Thus, we can state that the contents of geometry in school textbooks comprise, according to their purpose, a structure based on situations, memorization, understanding of concepts, construction of concepts and rules, language, procedures, applicability, propositions and arguments.

Finally, it is pertinent to highlight that the didactic analysis of school texts, where the variety of tools and constructs of Mathematics Education in geometry are put into play, allow the development of professional competencies in teacher training and the capacity for analysis and interpretation in students. At this point it would be possible to build bridges between these historical studies on geometry and its teaching, with critical perspectives.

The above does not indicate that a synthesis between two perspectives of analysis should be forced, since the objects are regularly different. However, they coincide in two ideals: they inscribe in a critical perspective that, anchored in the same tradition of modernity, is concerned with the current moment and what influences the learning environment and, secondly, the critical exercise implies a change from an analysis that allows to see it in the framework of a set of relationships with other objects, with forms of exercise of power, with discourses, among other aspects, that will allow the inclusion of new technologies, enabling advances in the categories that structure the mathematical texts for the teaching of geometry.

OBJECTIVES

- Analyze the contents of geometry textbooks.
- Identify categories of analysis for textbooks or school textbooks.
- Select the most relevant textbooks.

METHODOLOGY

In this research, the design of content analysis was used, taking into account that the main axis of the research is to look at how mathematics texts, in which geometry is immersed, have been becoming a great boom for the teacher from their planning and for the student as a fundamental element for research and self-guidance. According to Berelson: "The analysis of content is a research technique for the objective, systematic and quantitative description of the manifest content of communication" (1952, p. 1). In this sense, the research developed was based, as dictated by content analysis, on reading, which allowed undermining and collecting information without losing sight of the rigor of the scientific method, which requires it to be systematic, objective, replicable, and valid.

This research follows the 5 steps for the content analysis process proposed by Abela (2002), which are:

1. Determine the object or subject of analysis.
2. Determine the coding rules.
3. Determine the category system.
4. Check the reliability of the coding-categorization system.
5. Inferences

CONCLUSIONS

It is important that, although textbooks in their evolution continue to be used in current education, technologies applied to education are providing the ideal conditions for the development of new tools for the teaching of geometry. It would be considered as an advance and use of the technological era, that future research and applications are focused on digital textbooks since they have the ability to solve and improve the deficiencies that can be found in current books as far as the visualization of geometry is concerned. Therefore, it is necessary that digital textbooks are interactive and dynamic, so that they can provide students and teachers with digital geometric environments according to their needs. It is suggested that the development of these books is not only an innovative creation, but also that its content is more assertive, with no faults or shortcomings for learning in this area.

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