Recibido: 10 Abr., 2023 | **Aceptado:** 15 May., 2024 | **Publicado:** 29 May., 2024

RESEARCH ARTICLE

Semiotic transformations to analyze interpretation and representation problem solving strategie

Las transformaciones semióticas para analizar estrategias de solución de problemas de interpretación y representación*

Transformações semióticas para analisar estratégias de resolução de problemas para interpretação e representação.

DOI: https://doi.org/10.21803/penamer.17.33.602

Jesús David Berrio Valbuena
http://orcid.org/0000-0002-4014-5322
PhD. (e) en Educación Matemática Universidad Pedagógica Experimental Libertador, Docente tiempo completo Universidad del Atlántico,
jberriovalbuena@mail.uniatlantico.edu.co

Sonia Valbuena Duarte
https://orcid.org/0000-0003-3667-1087
PhD. (c) en Matemáticas,
Docente Universidad del Atlántico.
soniabalbuena@mail.uniatlantico.edu.co

Rafael Sánchez Anillo
https://orcid.org/0000-0002-8924-055
PhD. en Educación, Docente tiempo completo Universidad del Atlántico.
rafaelsanchez@mail.uniatlantico.edu.co

Abstract

Introduction: The theory of registration of semiotic representations establishes that the use of sign systems, which allow the cognitive activities of formation, conversion and treatment, is essential to have access to mathematical objects or processes. **Objective:** This research aims to analyze the strategies used by eleventh-grade students to solve problems of interpretation and representation through treatment and conversion activities. **Methodology:** A questionnaire was selected with questions of the type Saber tests, in which the competence of interpretation and representation is evaluated. Also, to show what each student does and says when answering the questions, video recordings and transcriptions are made. **Results**: An *a priori* analysis is proposed per question, to analyze and interpret the cognitive treatment and conversion activities used by the students, as well as an *a posteriori* analysis where the results obtained are analyzed and the hypotheses are verified. **Conclusions:** It shows how students associate the significant units of a problem posed in a representation register to another register, to find either a contrast of the information or a relationship in the information, taking into account the semantic correspondence and semantic univocity.

Keywords: Conversion; Interpretation and representation; Treatment; Semiotic representations.

Resumen

Introducción: La teoría de registro de representaciones semióticas establece que el uso de sistemas de signos que permiten las actividades cognitivas de formación, conversión y tratamiento son esenciales para tener acceso a los objetos o procesos matemáticos. **Objetivo:** Esta investigación tiene como objetivo analizar estrategias utilizadas por estudiantes de grado 11 para resolver problemas de interpretación y representación mediante las actividades de tratamiento y conversión. **Metodología:** Se seleccionó un cuestionario con preguntas del tipo pruebas Saber en el que se evalúa la competencia de interpretación y representación. Asimismo, para poner de manifiesto qué hace y dice cada estudiante cuando responde las preguntas, se realizan videograbaciones y transcripciones. **Resultados:** Se plantea un análisis *a priori* por pregunta para analizar e interpretar las actividades cognitivas de tratamiento y conversión empleadas por los estudiantes, así como un análisis *a posteriori* para analizar los resultados obtenidos y verificar las hipótesis planteadas. **Conclusiones:** Se muestra cómo los estudiantes asocian las unidades significantes de un problema planteado en un registro de representación a otro registro, para encontrar, bien un contraste de la información, bien una relación en la información, en atención a la correspondencia y la univocidad semántica.

Palabras clave: Conversión; Interpretación y representación; Tratamiento; Representaciones semióticas

¿Cómo citar este artículo?
Berrio; J., Valbuena; S. y Sánchez; R. (2024). Las transformaciones semióticas para analizar estrategias de solución de problemas de interpretación y representación. *Pensamiento Americano, e#602. 17*(33), 1-13. DOI: https://doi.org/10.21803/penamer.17.33.602

** Los términos clave han sido recuperados a partir del Tesauro Europeo de la Educación (2023). https://www.vocabularyserver.com/tee/es/index.php?tema=2173*

RESEARCH ARTICLE

Resumo

Introdução: A teoria de registro das representações semióticas afirma que o uso de sistemas de sinais que permitem as atividades cognitivas de formação, conversão e processamento é essencial para acessar objetos ou processos matemáticos. **Objetivo:** Esta pesquisa tem como objetivo analisar as estratégias utilizadas por alunos da 11ª série para resolver problemas de interpretação e representação por meio de atividades de processamento e conversão. **Metodologia:** Foi selecionado um questionário com perguntas do tipo teste Saber, no qual é avaliada a competência de interpretação e representação. Também foram feitas gravações e transcrições de vídeo para mostrar o que cada aluno faz e diz ao responder às perguntas. **Resultados:** Propõe-se uma análise *a priori* por questão para analisar e interpretar o processamento cognitivo e as atividades de conversão empregadas pelos alunos, bem como uma análise *a posteriori* para analisar os resultados obtidos e verificar as hipóteses apresentadas. **Conclusões:** É demonstrado como os alunos associam as unidades de significado de um problema apresentado em um registro de representação a outro registro, a fim de encontrar um contraste de informações ou uma relação nas informações, com atenção à correspondência e à univocidade semântica.

Palavras-chave: conversão; interpretação e representação; processamento; representações semióticas.

Introduction

At present, learning mathematics is one of the fundamental learning processes in elementary education, given the instrumental nature of these contents. In this context, Peña Ubarne and Berrío Valbuena (2022, 2023) and Valbuena Duarte et al. (2020) explain that understanding the difficulties in learning mathematics has become a concern for a large number of professionals dedicated to the world of education. This is especially relevant if we consider the high percentage of failure in these contents among students who complete compulsory schooling. Students show weaknesses when asked to express, interpret, represent and evaluate mathematical ideas, this is proven by the Colombian Institute for the Evaluation of Education (ICFES) and the Program for International Student Assessment (PISA), exposing the worrying level of performance in the area of mathematics in which there is greater concentration in the minimum and insufficient levels, placing 70% of students in these two levels (ICFES, 2018; Sánchez, 2023; Subdirección de Calidad y Pertinencia Dirección de Evaluación de la Educación Secretaría de Educación del Distrito, 2014).

Most of the difficulties are specifically located in problems in which it is necessary to convert from the natural language register to the algebraic register or from the graphic register to the verbal register, as shown by different studies: Berrío Valbuena et al. (2020), Duval (1991, 1999, 2000, 2004, 2006, 2011), Iori (2017), Kataoka et al. (2022), Lemos Florez and Herrera Ruiz (2015), McGee and Martínez-Pla- nell (2014), Morales Martínez (2013), Ospina García (2012) and Valbuena Duarte et al. (2021). These authors present a broad description of some of the difficulties that students have in interpreting important ideas when solving a problem situation.

To address these and other difficulties encountered in the interpretation and representation of written problem situations both in natural language and in other representation registers, the need arises to study from a semiotic point of view the activities of processing and converting written mathematical problems into different representation registers.

The theory of registration of semiotic representations provides elements that allow analyzing the solution processes used in the resolution of a problem situation in mathematics, the transformations and the role they play in the understanding of these (Berrío Valbuena et al., 2020; Charris and Muñoz, 2019). According to the above, the purpose of this research is to analyze the strategies used by 11th grade students to solve problems of interpretation and re-presentation, as posed in the Saber tests, through the activities of processing and conversion of semiotic representations, in relation to the criteria of semantic correspondence congruence and semantic univocity.

Theoretical framework

The choice of a particular register of representation of a mathematical object or a given piece of information is often key to its understanding. The following are the types of representation registers, among which are the verbal register or natural language, numerical register, figural-iconic register, tabular register, algebraic register, graphic register and geometric register. Learning

The learning of mathematics includes an analysis of cognitive processes such as interpretation, these processes require the use of different registers of representation from those of natural language, whether algebraic, geometric or graphic.

For Duval (1999, 2000, 2011), a semiotic system, i.e., a sign system and a representational system are different things, so that, for a semiotic system to be a representational system, it must allow the following three cognitive activities:

- Formation: It is a resource of signs to actualize the look of an object or representations in a semiotic register, it serves to express a mental representation or to evoke a real object.
- Treatment: The transformation of an initial representation into a final representation of an issue or problem in the same register of representation. Treatment is a strictly internal transformation in a register, i.e., the system of signs in which the representation is expressed is not changed; therefore, it occurs with the only rules proper to that semiotic system.
- Conversion: It is the transformation of the representation of a mathematical object or a given information from one semiotic register to another. A treatment moves in the same register of representation; the opposite occurs in conversion, which moves from one register to another.

Between conversion and processing it is necessary to specify that cognitively they are independent of each other, although mathematically the former depends on the latter. Therefore, the conversion of a representation is the first step in the understanding of a mathematical object or process and, therefore, in the learning of mathematics.

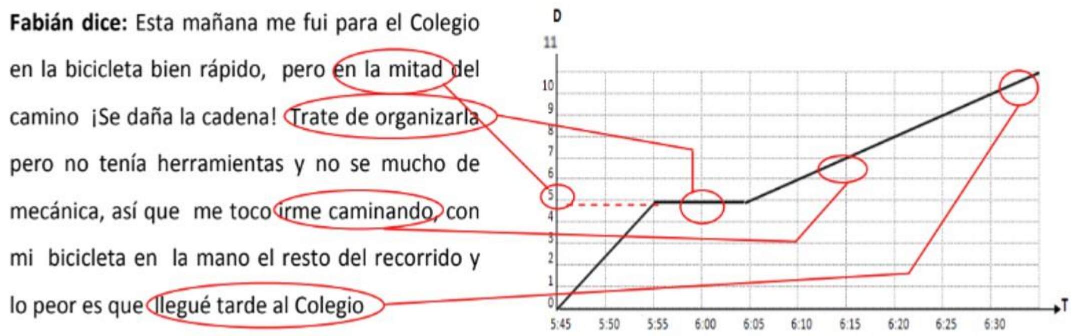
For Duval (2006), changing the representation of objects or mathematical relations from one semiotic system to another is always a cognitive leap. Unlike processing, there are no rules or basic associations, as between words and images in everyday language, for this type of representation transformation, the conversion is therefore not reduced to encoding.

The semantic correspondence of the signifying elements is defined by Duval (1999) as the association of each single signifying unit of the register of departure representation with each of the elementary signifying units of the register of arrival representation. And terminal semantic univocity is defined as the assignment to each elementary signifying unit in the register of the representation of departure a single elementary signifying unit in the register of the representation of arrival. Significant units are understood as the elements or characteristics that the different variables can take in each register of representation.

In the following example (Figure 1), the relationship of the signifying units in the natural language register and the graphic register can be observed, as well as their semantic correspondence and semantic univocity.

Figure 1.

Significant units and correspondence between the verbal and graphic registers.



Source: Adapted from Ospina García (2012).

This study intends to show how students associate the significant units of a problem posed in a representation register to another register, in order to find either a contrast of the information or a relationship in the information. Being more specific, it will analyze in a practical way how students transform the information that is represented to another representation, which allows them to have a quick and correct answer at the moment of extracting relevant information from the problem situation posed in the starting register (significant units) and how they relate this information to the arrival register (semantic correspondence), that is, it is expected that students will be able to interpret the information provided in the situations posed and make transformations in the different registers of semiotic representation.

Methodology

This study has a qualitative approach (Martínez, 2006) and follows a clinical research strategy in mathematics education (Camargo Uribe, 2019), "because it facilitates the construction of research designs of the empirical world, allowing researchers to connect their theoretical assumptions with specific ways of obtaining information to analyze it" (p. 2). An *a priori* and *a posteriori* analysis approach was implemented. For Orús and Pitarch (2004), *a priori* and *a posteriori* analyses provide more complete and nuanced information on the reality of information processing. *A priori* analyses will allow the establishment of hypotheses about the strategies that the individual who solves the problem posed would use, i.e., it allows the *a priori* relationships between the variables to be revealed according to their characteristics and contrasted with the result of the experimentation. On the other hand, the *a posteriori* analyses will determine to what extent the results of the experimentation can be explained in terms of the criteria of the questions established in the *a priori* analyses.

Collection of information

In this study, a questionnaire was administered to three 11th grade students from a public educational institution in Barranquilla. Questionnaires are sets of questions used to study a problem by pointing out certain aspects of it; these questions can be closed or open-ended (see Figure 1).

nández Sampieri et al., 2010). Closed-ended questions establish delimited response options, they represent the possibilities of response choice for the participants and, therefore, the categories will also be limited. On the other hand, open-ended questions do not establish limits to the response, so the number of response categories is indeterminate and is susceptible to variation according to the population that responds. Given the above considerations, questions were selected according to the characteristics of the Saber tests designed by ICFES. The questionnaire was divided into three sections, each containing three questions, and the analysis of the solution strategies of a closed question will be shown.

On the other hand, in order to show what each student does and says, video recordings and transcriptions are used as data collection instruments.

Video recordings

For Rich and Hannafin (2009), video recordings, being audiovisual tools, allow researchers to document the conceptual, procedural and strategic contents of students or to evaluate aspects such as communication skills, interpretation, analysis, creativity and innovation. Among its main characteristics are its expressive potential, i.e., in which the student explains the work done and demonstrates the knowledge acquired. In this way, the video recordings were used in the solution of the problem situations in a personalized way, with the objective of recording the processes of treatment and conversion of the semiotic representations made by each student.

Transcriptions

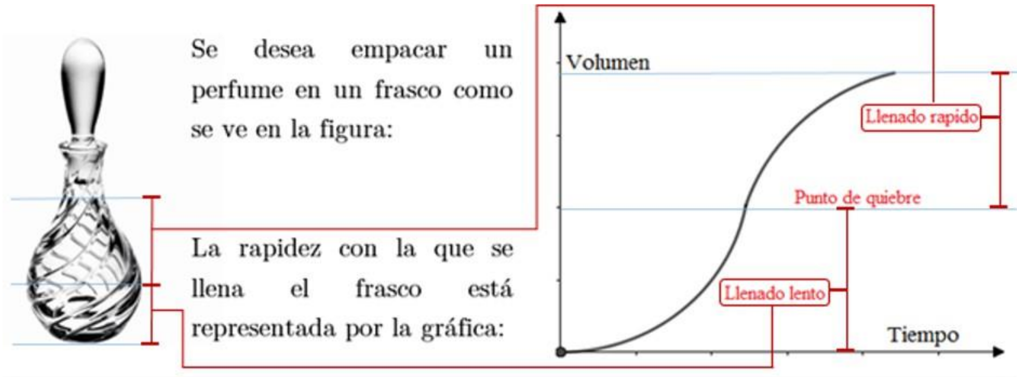
According to Sánchez Gómez and Revuelta Domínguez (2005) and Strauss (1987), when the data in an investigation have been recorded using audiovisual media, their transcription is a necessary step for their interpretation, since they allow for ample information about the nature and purpose of a study. Thus, the transcriptions made on the video recordings in this research made it possible to analyze the solution strategies used by the students in the identification of the significant units and the processes they carried out when matching them in the different representation registers, providing a broad view of the objective of this study.

Results

The following closed question was proposed during one of the sections of the questionnaire applied, it is expected that the students do not have problems in establishing a direct relationship between the intervening variables in the figural registers (starting register) with those of the graphic register (arrival register), delimiting each significant unit of the starting register and making it correspond with its corresponding unit in the arrival register, carrying out the cognitive activity of conversion. The aim is to determine whether students can move between representation registers (Figure 2).

Figure 2.

Conversion of the figurative register to the graphic register.



Source: Own elaboration.

A priori analysis hypothesis

The student, observing the drawing, realizes that the filling of the perfume bottle is divided into two intervals or moments, which are separated by a break point.

In the first interval, the filling goes from a small circular surface to a large circular surface, which means that with the passage of time the volume will rise slowly, due to the shape of the perfume bottle in this interval.

Also, after the end of the first interval and the beginning of the second, the volume goes from rising slowly to going fast, because it goes from a large circular surface to a small circular surface; therefore, at the beginning of the graph, the filling is slow while after the break point it is fast.

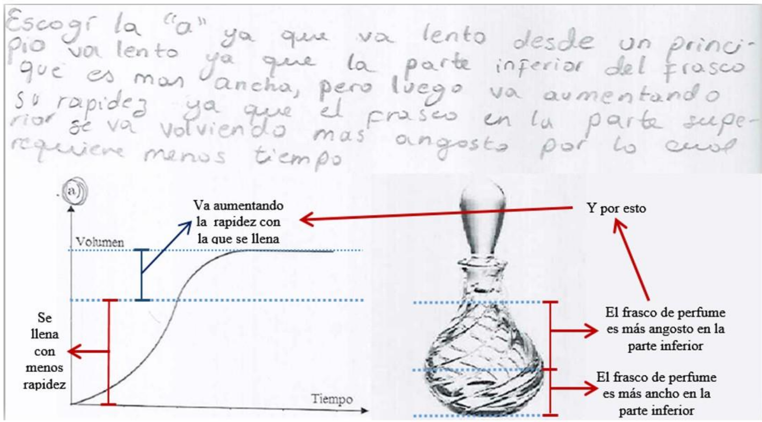
Ex-post analysis

The results obtained from the implementation of the proposed questionnaire will be described through a detailed analysis, highlighting the main findings in the solution strategies of each of the three participating students.

Student 1 says: "As the figure of the perfume is wider at the bottom so it will fill less quickly and then it increases in speed because it becomes narrower at the top, so it takes less time to fill" (Figure 3).

Figure 3.

Student response 1



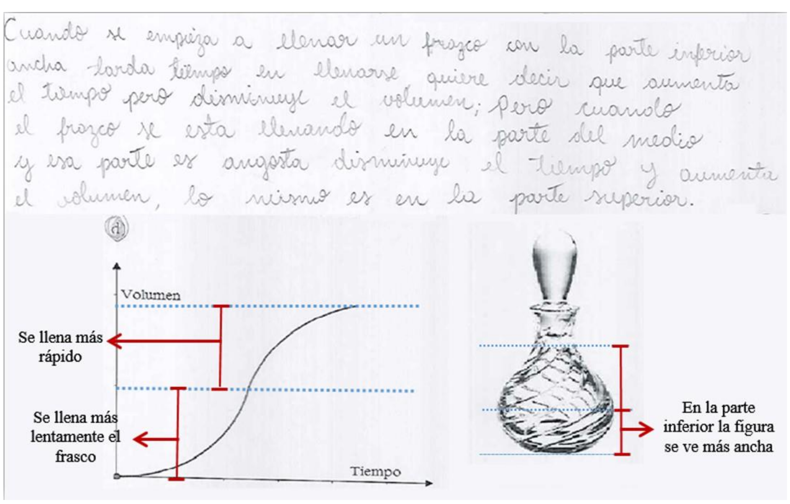
Source: Own elaboration.

Student 1 identifies the significant units of the first and second moment: "less speed", "is increasing its speed", but does not identify or point out the break point (blue dotted line). In addition, he does not manage to convert the signifying unit of the second moment to the graphic register, then he places in correspondence the signifying unit of the first moment, but he gets confused when segmenting the signifying unit of the second moment to the graphic register, which is indicated with blue, he also fails to segment the break point in both registers; on the other hand, since the signifying unit of the second moment does not have semantic correspondence because in both registers they do not mean the same thing, therefore, there is no semantic univocity.

Similarly, student 2 says: "In the lower part of the figure it looks wider, so it takes longer to fill, but its volume will be smaller, that is, the jar fills more slowly, but in the middle part it takes less time and has more volume, that is, it fills faster, the same in the upper part" (figure 4).

Figure 4.

Student response 2



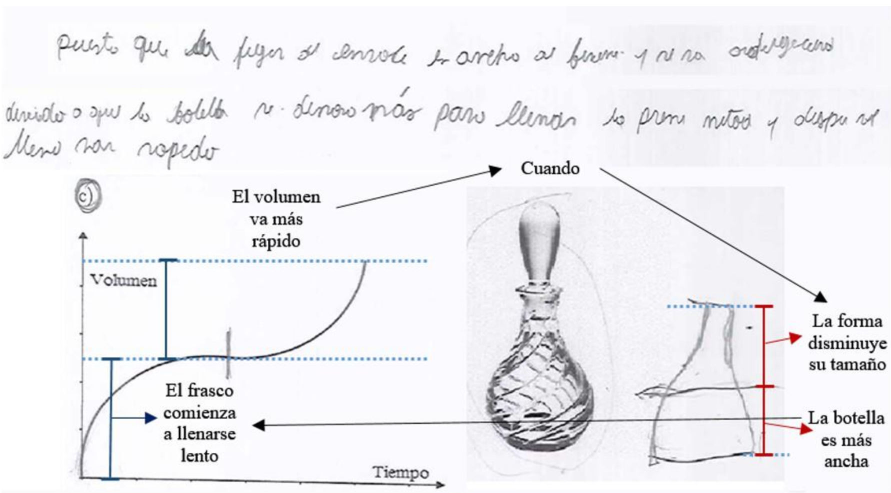
Source: Own elaboration.

Student 2 identifies the significant units, realizes that, because of the shape of the lower part, the jar will fill slowly, contrary to the upper part, which will fill quickly; however, he has some confusion in the interpretation of the information because he mentions that the time increases or decreases, when the time elapses steadily, this type of difficulty being an incorrect use of language. After making the conversion, he takes the significant units of the starting register and puts them in correspondence with those of the arrival register. On the other hand, there is semantic univocity because each signifying unit in the register of departure corresponds to a unit in the register of arrival.

For his part, student 3 says: "Taking into account the figure of the bottle then at the beginning it begins to fill slowly until it reaches the point in the bottle where it begins to close by the lid, then the graph of the bottle would be divided into 2 moments, which would be when it takes time to fill which would be when the bottle is wide and then when the volume advances faster which would be when the shape decreases in size" (figure 5).

Figure 5.

Student response 3



Source: Own elaboration.

Student 3 identifies the significant units of the first and second moments, which are where "the filling is slow" and "it advances fast", as well as identifies the break point that divides the exercise into two moments, but does not make the proper conversion of these in the graphic register, likewise, there is no semantic correspondence between the significant units of the first and second moments, as a result of inverting the order in the segmentation; Consequently, there is no semantic univocity between the signifying units of both registers, because the two signifying units found in the figural register and the units expressed in the graphic register do not mean the same thing in both representations.

Discussion

Global ex-post analysis

In this section, a general analysis of what happened in the previous process is provided in accordance with two congruence criteria and it will be verified whether the hypotheses proposed by the researchers were fulfilled. The following acronyms will be considered for this analysis:

CS: semantic correspondence; US: semantic univocity; RG: graphic register; RF: figural register.

CS: No, students show difficulty segmenting the signifying units of the starting register (RF) with the signifying units of the target register (RG).

US: No, students are able to identify the significant units of the record of departure (RF), but have difficulty matching them to the record of arrival (RG).

At the moment of converting the figurative register to the graphic register, two of the students identify the significant units of the starting register (RF), but it is difficult for them to match them with the significant units of the target register (RG), making an erroneous solution to the problem posed, because the significant units found in the figurative register and the units expressed in the graphic register do not mean the same thing in both representations.

Conclusions

The general objective of this research was to analyze the strategies used by 11th grade students to solve problems of interpretation and representation through the activities of processing and conversion of semiotic representations. In the results obtained, it is observed that students used as a strategy to solve the situation posed in the figurative register to the graphic register the identification of the significant units, to then put them in correspondence in the other registers of representation, although it should be noted that they excluded some significant units, which led them to obtain erroneous results.

According to Duval (2004), learning centered on the transformation of some representations into others allows obtaining new information and properties, extracting new knowledge from the objects, ideas and concepts represented, the cognitive activities of processing and conversion used by the students allowed them to understand and interpret the information presented.

Two major difficulties were found associated with the solution of closed questions of the Saber test type that evaluate interpretation and representation competencies:

- Students have difficulties associating information from the figurative register with the graphic register, because they are instructed in the classroom to solve situations in an algorithmic way. And at the moment of using the different representations, it is difficult for them to coordinate and integrate the information.

- When students face the Saber tests, they have trouble getting an answer quickly, knowing that they have one or two minutes to answer each question and that the time implemented to answer this question far exceeds the standard response time.

The findings of this work offer an empirical contribution to the formation of competencies of high school students in the approach and resolution of multiple-choice questions, as well as the identification of the use of alternative teaching and evaluation models to the traditional approach to the study of mathematics (Berrío Valbuena et al., 2017). Likewise, it allows recognizing the importance of the postulates of the theory of semiotic representation registers for the development of teaching, learning and evaluation strategies that meet the challenges proposed by the results of national and international standardized tests (Valbuena Duarte et al., 2021).

References

Berrío Valbuena, J. D., González Roca, A., Meriño Mendoza, V. A. y Valbuena Duarte, S. (2020). ¿Cómo interpretan noticias sobre el covid-19 los estudiantes de un programa de formación de profesores de matemáticas? En J. Solorzano Movilla y D. Suárez López (comps.), *Miradas y perspectivas de la educación matemática desde la formación, la inclusión y la tecnología* (pp. 146-173). Sello Editorial Coruniamericana. https://repositorio.americana.edu.co/entities/publication/7cc1477e-c900-489e-9ba4-489c93d-9f2b3

Berrío Valbuena, J. D., Valbuena Duarte, S. y Sánchez Anillo, R. (2017). Simulación de composición de simetrías en Geogebra para simplificar la teoría de los diamantes en la práctica del billar. *Revista Pensamiento Americano, 10(19),* 55-67. https://doi.org/10.21803/pensam.v10i19.29

Camargo Uribe, L. (2019). Estrategias cualitativas de investigación en educación matemática [conferencia]. *XV Conferencia Interamericana de Educación Matemática, Medellín, Colombia.* https://conferencia.ciaem-redumate.org/index.php/xvciaem/xv/paper/viewFile/1061/568

Charris, M. y Muñoz, C. (2019). *Transformaciones semióticas en la resolución de problemas de interpretación y representación en estudiantes de undécimo grado* [tesis de grado, Universidad del Atlántico].

Duval, R. (1991). Structure du raisonnement déductif et apprentissage de la demonstration. *Educational Studies in Mathematics, 22*(3), 233-261. https://doi.org/10.1007/BF00368340

Duval, R. (1999). *Los problemas fundamentales en el aprendizaje de las matemáticas y las formas superiores del desarrollo cognitivo.* Universidad del Valle.

Duval, R. (2000). Ecriture, raisonnement et découverte de la démonstration en mathématiques. *Recherches en didactique des mathématiques, 20*(2), 135-170.

Duval, R. (2004). *Semiosis y pensamiento humano: Registros semióticos y aprendizajes intelectuales.* Universidad del Valle.

Duval, R. (2006). *Un tema crucial en la educación matemática: La habilidad para cambiar el registro de representación.* La Gaceta de la Real Sociedad Matemática Española.

Duval, R. (2011). *El aprendizaje del álgebra y el problema cognitivo de la designación de objetos.* Universidad del Valle.

Hernández Sampieri, R., Fernández Collado, C. y Baptista Lucio, M. del P. (2010). *Metodología de la investigación.* McGraw-Hill.

Hoyos Estrada, S. . (2019). La influencia de los discounter en el cambio de hábitos de consumo de los colombianos. *Ad-Gnosis, 8*(8). https://doi.org/10.21803/adgnosis.v8i8.366

Instituto Colombiano para la Evaluación de la Educación. (2018). *Guía introductoria al diseño centrado en evidencias.* https://www.icfes.gov.co/documents/39286/443287/Guia+introductoria+al+Diseno+Centrado+en+Evidencias.pdf

Iori, M. (2017). Objects, signs, and representations in the semio-cognitive analysis of the processes involved in teaching and learning mathematics: A Duvalian perspective. *Educational Studies in Mathematics, 94*(1), 275-291. https://doi.org/10.1007/s10649-016-9726-3

Kataoka, Y., Carvalho, A. y Borim, C. (2022). The concept of chance in early childhood education: An analysis from the perspective of the register of semiotic representations. *Statistics Education Research Journal, 20*(2), 1-19. https://doi.org/10.52041/serj.v20i2.318

Lemos Florez, K. y Herrera Ruiz, N. D. (2015). *Análisis del proceso de conversión de problemas escritos en lengua natural a un sistema de ecuaciones lineales* [tesis de grado, Universidad del Valle]. https://bibliotecadigital.univalle.edu.co/server/api/core/bitstreams/57d54342-75c8-4acd-9ca5-dd238c6d493f/content

Martínez, M. (2006). La investigación cualitativa (síntesis conceptual). *Revista de Investigación en Psicología, 9*(1), 123-146

McGee, D. L. y Martínez-Planell, R. (2014). A study of semiotic registers in the development of the definite integral of functions of two and three variables. *International Journal of Science and Mathematics Education, 12,* 883-916. https://doi.org/10.1007/s10763-013-9437-5n

Morales Martínez, Z. E. (2013). *Análisis de las transformaciones de las representaciones: Semióticas en el estudio de la función logarítmica en la educación escolar* [tesis de maestría, Universidad Católica del Perú]. https://tesis.pucp.edu.pe/repositorio/bitstream/handle/20.500.12404/4639/MORALES_MARTINEZ_ZENON_ANALISIS_ESCOLAR.pdf?sequence=1&isAllowed=y

Orús, P. y Pitarch, I. (2004). *Análisis didáctico de un cuestionario en el marco de la teoría de las situaciones.* En Actas del VIII Simposio de Investigación en Educación Matemática.

Ospina García, D. (2012). Las representaciones semióticas en el aprendizaje del concepto función lineal [tesis de maestría, Universidad Autónoma de Manizales]. https://repositorio.autonoma.edu.co/bitstream/11182/477/4/Representaciones_semi%c3%b3tica_aprendizaje_concepto_funcional_lineal.pdf

Peña Ubarne, Z. F. y Berrío Valbuena, J. D. (2022). Creencias acerca de las matemáticas y su aprendizaje en la modalidad virtual en tiempos de pandemia de covid-19 en estudiantes de bachillerato. *Números: Revista de Didáctica de las Matemáticas, 100*(1), 7-24. https://redined.educacion.gob.es/xmlui/bitstream/handle/11162/242190/Documento%201.pdf?sequence=1&isAllowed=y

Peña Ubarne, Z. F. y Berrío Valbuena, J. D. (2023). Desmitificar las matemáticas: Cómo las creencias afectan el proceso de enseñanza-aprendizaje y qué estrategia podemos seguir. En A. M. Rosas Mendoza (ed.), *Avances en matemática educativa el alumno desde la teoría Nº 14* (pp. 33-47). Lectorum. https://www.cicata.ipn.mx/assets/files/cicata/ProME/docs/Archivos/ProgramaEditorial/AvaME/AMEANo11.pdf

Rich, P. J. y Hannafin, M. (2009). Video annotation tools: Technologies to scaffold, structure, and transform teacher reflection. *Journal of Teacher Education, 60*(1), 52-67. https://doi.org/10.1177/0022487108328486

Sánchez G., V. (2023, 2 de enero). Colombia se ha mantenido en últimos lugares de prueba Pisa en recientes ediciones. *La República.* https://www.larepublica.co/globoeconomia/colombia-se-ha-mantenido-en-los-ultimos-puestos-de-la-prueba-pisa-durante-ultimas-ediciones-3517806.

Sánchez Gómez, M. C. y Revuelta Domínguez, F. I. (2009). El proceso de transcripción en el marco de la metodología de investigación cualitativa actual. Enseñanza & Teaching: *Revista Interuniversitaria de Didáctica, 23,* 367-386.

Strauss, A. (1987). *Qualitative analysis for social scientists.* Cambridge University Press.

Subdirección de Calidad y Pertinencia Dirección de Evaluación de la Educación Secretaría de Educación del Distrito. (2014). Análisis descriptivo y pedagógico de los resultados de las pruebas Saber 3º, 5º y 9º (2009, 2012 y 2013) y Saber 11 (2014) https://repositoriosed.educacionbogota.edu.co/server/api/core/bitstreams/40de8f4b-d100-4523-9253-77dbde1230df/content

Valbuena Duarte, S., Muñiz Márquez, L. E. y Berrio Valbuena, J. D. (2020). El rol del docente en la argumentación matemática de estudiantes para la resolución de problemas. *Revista Espacios, 41*(9), 1-15. https://w.revistaespacios.com/a20v41n09/a20v41n09p15.pdf

Valbuena Duarte, S., Tamara Gutiérrez, Y. y Berrío Valbuena, J. (2021). Intervención didáctica tecnológica para el estudio de las secciones cónicas basada en el potencial semiótico. *Formación Universitaria, 14*(1), 181-194. http://dx.doi.org/10.4067/S0718-50062021000100181

2024, Vol. 17(33) 1-13. ©The Author(s)
Reprints and permission: www.americana.edu.co
http://publicaciones.americana.edu.co/index.php/pensamientoamericano/index